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Economic Theory

The structure of authority, federalism, commitment and economic growth*

Ronald A. Edwards

The Institute of Economics and the Institute of History and Philology, Academia Sinica,
Taipei, Taiwan, REPUBLIC OF CHINA (e-mail: redwards@pluto.ihp.sinica.edu.tw)

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Summary. In a neoclassical growth model with many regions and a mobile factor, two federal arrangements are considered. In the first federal arrangement the central government chooses a uniform tax policy, whereas in the second each regional government chooses its own tax policy. The main result is that the first federal arrangement leads to high tax rates and economic stagnation, whereas the second leads to low tax rates and economic growth. This result stems from a time consistency problem. The lack of tax competition forces a time consistency problem on the central government under the first federal arrangement. In contrast, regional tax competition acts as a commitment device under the second federal arrangement. The fundamental feature in the environment that gives rise to different abilities of the state to commit is the different structure of authority within the state.

Keywords and Phrases: Federalism, Institutions, Commitment, Time consistency, Growth theory, Game theory, Political economy.

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1 Introduction

Motivated in large part by the pioneering work of Riker (1964), there has been increased interest in federalism and its effects on economic performance. Many

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scholars argue in various ways that federalism has first-order effects on economic development. This paper makes a theoretical contribution to this literature by formalizing an environment in which federal arrangements with different structures of authority lead to different equilibrium growth rates.

Most closely related to the main theme of this paper is recent research on federalism that addresses a fundamental problem in government: how to design the state so that it can commit not to expropriate in the future. This classic problem is called the time consistency problem in the macroeconomics literature.[1] Some recent work on this issue includes North and Weingast (1989), Weingast (1995) and Greif (1998). North and Weingast (1989) argue that the separation and balance of powers within the central government are main checks. Weingast (1995) argues that by challenging policy transgressions, citizens keep the government in check. In the historical case of Genoa, Greif (1998) suggests that a balance of power among clans within government limits intrusive policies. This paper formalizes a different mechanism: regional tax competition that can commit the state not to expropriate in the future.

I consider two federal arrangements in a standard neoclassical growth model with many regions and a mobile factor, human capital, which is the key factor to economic growth. The fundamental difference between the two federal arrangements is their different structures of policy authority within the state. In the first federal arrangement, the central government chooses a uniform tax policy for all regions. In the second federal arrangement, each regional government non-cooperatively chooses its own regional tax policy. Due to the mobile factor, regional governments face tax competition under the second federal arrangement. In contrast, the central government, which chooses a uniform tax policy, faces no tax competition under the first federal arrangement. The main result is that different choices of economic policy arise in equilibrium under the two different federal arrangements, which lead to different rates of human capital investment and thus different growth rates. This result stems from a time consistency problem. Under the first federal arrangement, the lack of regional tax competition forces a time consistency problem on the central government. While under the second federal arrangement, regional tax competition acts as a commitment device. In this particular environment, the devolution of tax policy authority from the central government to regional governments introduces regional tax competition, which removes the time consistency problem and commits the state not to expropriate in the future. The fundamental feature in the environment that gives rise to different abilities of the state to commit is the different structure of authority within the state.

This paper is also related to branches of research within growth theory including human capital, political elites and total factor productivity as well as local public finance. Specifically, there is a large body of related literature on human capital, which is largely due to the pioneering work of Romer (1986) and Lucas (1988), and more recently Lucas (2002). Recent work by Acemoglu and Robinson (2000, 2002) has introduced political elites into a growth theory framework, offering a

[1] See Kydland and Prescott (1977). Related recent research on government in macroeconomics includes Persson and Tabellini (2000), which explores various aspects of policy decision.

complementary approach to the general theme of this paper. While the focus of this paper is on the discovery of new technologies, another related tradition within growth theory emphasizes the adoption of existing technologies.[2] This model extends many tax competition results, which are related to a body of literature on local public finance stemming from Tiebout (1956) (such as Kehoe (1989)), into a dynamic general equilibrium framework.

The remainder of this paper is organized as follows. In Section 2, the environment is described. Section 3 introduces groups in the dynamic game, federal arrangements and equilibrium definitions. Section 4 presents the main result of existence and characterization of equilibrium under both federal arrangements, Section 5 concludes and the Appendix gives all proofs.

2 Model environment

This section describes regions, preferences, endowments, land ownership, technology, the groups in and the timing of the sequence of actions in the dynamic game. Before specifying the environment, however, a brief comment about notation and dynamic games is in order.

Introducing game theoretic elements into growth theory presents many notational problems. One method to simplify the presentation of the desired economics is not to present the model in its most general framework. For example, symmetry can be exploited to drastically reduce notation. In this model groups play a dynamic game. In a model with measure one of regions, only regionally symmetric equilibria are considered. This could be modeled formally as an anonymous game. A state space of type defining characteristics could be given for each period. A measure, defined on the Borel sigma algebra of the state space and describing the measure of regions with a set of types, could be used to keep track of the relevant groups in the regions. However, since only regionally symmetric equilibria are considered, this would amount to keeping track of a Dirac measure across time. In this paper the players and their choices are not formally introduced in game theoretic language, but rather more closely follow the dynamic general equilibrium tradition. Many opportunities to simplify notation in the environment will be exploited, including using some obvious equilibrium conditions. This approach reduces notation and illustrates the desired economics more clearly.

There is measure one of ex ante identical regions, each of which has an initial population of measure two and a land endowment of $\overline{L} = 1$. There is no population growth and each region has a maximum population limit of measure $M > 4$. There is no cost for either people or goods to move across regions. Only regionally symmetric equilibria are considered.

People live two periods and preferences over consumption are ordered by $U(c_t, c_{t+1}) = \log c_t + \beta \log c_{t+1}$, where $\beta > 0$. People are endowed with (ω, ω)

[2] This literature emphasizes the importance of cross-country differences in total factor productivity (TFP); see Klenow and Rodriguez-Clare (1997), Prescott (1998), and Hall and Jones (1999). One well established theory of cross-country TFP differences is that of vested interests blocking new technologies; see Olson (1982), Mokyr (1990), Krusell and Rios-Rull (1996), Parente and Prescott (2000), and Herrendorf and Teixeira (2003).

units of the consumption good and $(1, \varepsilon)$ units of labor services, where $0 < \omega$ and $0 < \varepsilon < 1$.[3] A young either becomes a student or worker. A date t student has a factory when old that can be operated anywhere or not at all. This factory is of type h_{t+1}, which is also his level of human capital and is a function of the date t aggregate knowledge.

Land in each region is owned by that region's old. Land does not depreciate and at the end of the period each region's land is inherited by the young in that region.

The consumption good is produced by both factory and household technologies, which are available in all regions. The factory technology requires human capital and labor services as inputs. A person with h_t units of human capital who hires $n_{f,t}$ units of labor services produces $B_f h_t^{\theta} n_{f,t}^{1-\theta}$ units of the consumption good $(0 < \theta < 1)$. A factory is the factory manager and his workers producing in a city away from the household and farm. The household technology requires both land and labor services as inputs. The application of $n_{h,t}$ units of labor services on l_t units of land produces $B_h l_t^{\phi} n_{h,t}^{1-\phi}$ units of the consumption good $(0 < \phi < 1)$. A household firm is the labor and land used in household and farm production. Household firms, which are tied to land, are not mobile, whereas factories are mobile.

Human capital is accumulated from a learning externality, which is related to the aggregate stock of knowledge, A_t. One region's stock of knowledge, a_t, is the product of that region's measure of factory managers with the average factory manager's level of human capital, i.e. $a_t = z_{f,t}\overline{h}_t$. The aggregate stock of knowledge, A_t, is the sum of stocks of knowledge across regions. A date t young who becomes a student will, when old, have a level of human capital determined by the expression, $h_{t+1} = \Phi(A_t) = \max\{1, \gamma A_t\}$, where $\gamma > 1$. No current period consumption good is produced from learning. The initial stock of aggregate knowledge is normalized to one, i.e. $A_1 = 1$.

Output is subject to taxation, which includes expropriation. Tax rates are technology specific. Output from factories can be taxed at any rate, while household output, being relatively difficult to observe, has a maximum feasible tax rate of $\overline{\tau} < 1$. Let $\tau_t = (\tau_{f,t}, \tau_{h,t})$ denote the period t factory and household tax rate pair or tax policy. Feasible tax rates satisfy: $0 \leq \tau_{f,t} \leq 1$ and $0 \leq \tau_{h,t} \leq \overline{\tau} < 1$.

In this environment, within each period we can consider the players in this dynamic game as three groups: government(s), young and old. Under the first federal arrangement, the central government chooses a uniform tax policy for all regions. Under the second federal arrangement, each regional government non-cooperatively chooses its own tax policy. Governments are the only large players in the dynamic game and maximize current period tax revenue.[4]

[3] Note that a strictly positive amount of goods and labor services is assumed in order to ensure continuity of the budget correspondence in the proof of existence.

[4] Note that a period in this model is about 30 years. Also note that under the second federal arrangement regional governments are large only in their own region. They do not affect aggregate variables since there is a maximum population limit in the region. Under the first federal arrangement the central government can affect aggregate variables. The fundamental cause of these different effects is different distributions of authority within government.

There are young and old groups in the regions and borrowing and lending can occur within but not across generations. Each young group lives in a region and can allocate a fraction of its members to be students and the remainder to be workers. The young group maximizes its utility. Lotteries could be used to determine which young are allocated as chosen by the group. Given the convexity of preferences, however, all members of the group receive the same consumption plan so that lotteries are not formally required. In each region a young group is of some measure, which can be assumed to be one without loss of generality, so having one young group per region suffices. Each young group inherits the region's land at the end of the period. When old, this group maximizes its consumption and outcomes are determined competitively.[5] Thus each region will have a group of young and old each of measure one. Since only symmetric equilibria are considered we may restrict our attention to the representative region. Thus in this dynamic game the players consist of three groups: government(s), and the representative young and old groups.

In each period these groups play a three-stage game with the following choices. At stage 1, young groups assign members as student or worker. At stage 2, the date t government(s) chooses tax rates. At stage 3, factories move to the region with the lowest tax rate and outcomes are determined competitively. Note in stage 3 the wage rate will be the same in all regions.

3 Federal arrangements and equilibrium definitions

In each period two tax policies will be considered, $\tau_t = (\tau_{f,t}, \tau_{h,t})$ and $\overline{\tau}_t = (\overline{\tau}_{f,t}, \overline{\tau}_{h,t})$. These tax policies are interpreted as the tax policy in one region, τ_t, and the tax policy in all other regions, $\overline{\tau}_t$ (hereafter $\tau^t = (\tau_t, \overline{\tau}_t)$). In Section 3.1 a sequence of tax policies, $\tau = \{\tau^t\}_{t=1}^{\infty}$, is taken as given and the representative young and old groups' utility maximization problems are formally introduced. Given, τ, a τ-equilibrium is defined. In Sections 3.2 and 3.3 the first and second federal arrangements are described, the government(s) problem is formally presented and equilibrium is defined for both federal arrangements.

3.1 Young and old groups and τ-equilibrium

It will be useful to introduce some notation for aggregate variables. Given that each member of a young and old group will receive the same allocation, it follows that in each region all factories will be identical and similarly for household firms. Introducing measures of factories and household firms in each region and exploiting regional symmetry, one can then express economy-wide aggregate output as: $B_f A_t^{\theta} N_{f,t}^{1-\theta} + B_h N_{h,t}^{1-\phi}$, where $N_{f,t}$ and $N_{h,t}$ are aggregate factory and household labor demands; hereafter $N_t = (N_{f,t}, N_{h,t})$.[6] Note that since there is measure one

[5] Note that it is assumed that loan repayment within a group can be enforced even if one moves to another region.

[6] This simplification in notation requires the labor and land market clearing conditions, regional symmetry and the law of motion for the economy-wide stock of knowledge.

of regions and only regional symmetric equilibria are considered, the economy-wide aggregate output and the representative region's output will be equal.[7] To economize further on notation other aggregate variables will be denoted as follows: old group's labor supply to factories and households, $E_t = (E_{f,t}, E_{h,t})$; the young group's labor supply to factories and households and the number of young allocated to be students, $X_t = (X_{f,t}, X_{h,t}, X_{s,t})$; factory and household wages, $W_t = (W_{f,t}, W_{h,t})$; and generation t's consumption when young and old, $C^t = (C^t_t, C^t_{t+1})$. Next the representative old and young groups' utility maximization problems will be formally introduced.

Given A_1, W_1 and τ^1 the initial old maximize their utility:

$$\max_{C_1^0, N_1, E_1} \log C_1^0 \tag{1}$$

s.t.

$$\begin{aligned} C_1^0 \le\ & (1-\tau_{h,1})B_h N_{h,1}^{1-\phi} - W_{h,1}N_{h,1} \\ & + (1-\min\{\tau_{f,1}, \overline{\tau}_{f,1}\})B_f A_1^{\theta} N_{f,1}^{\theta} - W_{f,1}N_{f,1} \\ & + E_{h,1}W_{h,1} + E_{f,1}W_{f,1} + \omega, \\ E_{h,1} + E_{f,1} =\ & \varepsilon\ . \end{aligned}$$

Note that given the option of two factory tax rates, $\tau_{f,1}$ and $\overline{\tau}_{f,1}$, the factory manager will migrate to a region that offers the lowest tax rate. This feature represents the third stage of the period game. After regions choose their tax policies, factories migrate accordingly. Formally, this could be modeled as the factory manager's best response function given the tax policy $\tau^t = (\tau_t, \overline{\tau}_t)$. In the above formulation notation is reduced by simply using the smaller tax rate in the computation of after tax income. The formulation of generation t's problem is introduced in a similar way.

For a generation $t-1$ group that allocated $X_{s,t-1}$ young to be students, each having $\Phi(A_{t-1})$ units of human capital, given taxes and wages, after tax income from factory and household production can be expressed as:

$$\begin{aligned} & \pi_f(X_{s,t-1}, A_{t-1}, W_{f,t}, \tau^t, N_{f,t}) \\ & = (1-\min\{\tau_{f,t}, \overline{\tau}_{f,t}\})B_f(\Phi(A_{t-1})X_{s,t-1})^{\theta} N_{f,t}^{1-\theta} - W_{f,t}N_{f,t} \\ & \text{and} \quad \pi_h(W_{h,t}, \tau^t, N_{h,t}) = (1-\tau_{h,t})B_h N_{h,t}^{1-\phi} - W_{h,t}N_{h,t} \quad \text{respectively}\ . \end{aligned}$$

[7] Note that although the values of the representative region's and the economy-wide aggregate output agree, they are with respect to measures of different dimensions. Nonetheless, notation shall be abused here and capital letters will be used to represent both the representative region's and the economy-wide aggregate variables. The context of their use will determine their interpretation, with the exception of A_t, which will always be interpreted to be the economy-wide stock of knowledge.

Given A_t, W_t, W_{t+1} and τ^{t+1}, the generation t young group maximize lifetime utility:[8]

$$\max_{C^t, X_t, N_{t+1}, E_{t+1}} \log C_t^t + \beta \log C_{t+1}^t \tag{2}$$

s.t.

$$\begin{aligned} C_t^t &\leq W_{f,t}X_{f,t} + W_{h,t}X_{h,t} + \omega, \\ C_{t+1}^t &\leq \pi_f(X_{s,t}, A_t, W_{f,t+1}, \tau^{t+1}, N_{f,t+1}) \\ &\quad + \pi_h(W_{h,t+1}, \tau^{t+1}, N_{h,t+1}) \\ &\quad + E_{f,t+1}W_{f,t+1} + E_{h,t+1}W_{h,t+1} + \omega, \\ X_{s,t} &+ X_{f,t} + X_{h,t} = 1 \quad \text{and} \quad E_{f,t+1} + E_{h,t+1} = \varepsilon \ . \end{aligned}$$

Definition T. Given a sequence of tax policies,$\tau = \{\tau^t\}_{t=1}^{\infty}$, a τ-equilibrium is a non-negative sequence $C_1^0, \{A_{t+1}, W_t, X_t, E_t, N_t, C^t\}_{t=1}^{\infty}$ such that for all $t \geq 1$ the following four conditions hold:

1. Given A_1, W_1 and τ^1 the initial old group's problem is solved by C_1^0, N_1, E_1; i.e. it solves (1),
2. Given A_t, W_t, W_{t+1} and τ^{t+1} the generation t young group's problem is solved by $C^t, X_t, N_{t+1}, E_{t+1}$; i.e. it solves (2),
3. The consumption good, labor and land markets clear,
4. The young group's choices are consistent with the law of motion for the economy- wide stock of knowledge; i.e. $A_{t+1} = \Phi(A_t)X_{s,t}$.

3.2 First federal arrangement and definition of equilibrium

In this section the first federal arrangement is described and the central government's problem is fully specified. Lastly, equilibrium under the first federal arrangement is defined.

In the first federal arrangement the central government has the authority to choose tax policy. The central government chooses a uniform tax policy for all regions that maximizes current period aggregate tax revenue. In the first federal arrangement the regional governments enforce the central government's choice of policy and we otherwise abstract from the regional governments.

[8] Note that formally $X_{s,t}$ depends on τ^{t+1}. This function would be the best response function in the dynamic game. The best response function, $X_{s,t}(\tau^{t+1})$, would provide the value to be taken as given in the government problems. In order to reduce notation, these best response functions are not presented. Rather they are treated as fixed values. It should be pointed out that this minimal exposition is sufficient in order to prove and characterize equilibrium. If equilibrium values for these variables are proved to exist one can easily formally define a best response function where the desired equilibrium results by appropriately defining best responses for off equilibrium tax policies.

Given A_{t-1}, X_{t-1} and N_t the period t central government maximizes current tax revenue:[9]

$$\max_{\tau^t} \tau_{f,t} B_f (\Phi(A_{t-1})X_{t-1})^\theta N_{f,t}^{1-\theta} + \tau_{h,t} B_h N_{h,t}^{1-\phi} \tag{3}$$

s.t.

$$0 \leq \tau_{f,t} \leq 1, \quad 0 \leq \tau_{h,t} \leq \overline{\tau} < 1 \quad \text{and} \quad \tau_t = \overline{\tau}_t \ .$$

Since the central government picks a uniform tax policy for all regions, there is no possibility of factory migration. Lastly, the definition of equilibrium is given for the first federal arrangement.

Definition F1. Under the first federal arrangement, a symmetric equilibrium is a sequence of tax policies, $\tau = \{\tau^t\}_{t=1}^{\infty}$, and a non-negative sequence $C_1^0, \{A_{t+1}, W_t, X_t, E_t, N_t, C^t\}_{t=1}^{\infty}$ such that the later is a τ-equilibrium and such that for every $t \geq 1$, given A_{t-1}, X_{t-1} and N_t the period t central government's choice of τ^t solves (3).[10]

3.3 Second federal arrangement and definition of equilibrium

In this section the second federal arrangement is described and the regional government's problem is fully specified. Lastly, equilibrium under the second federal arrangement is given.

Under the second federal arrangement each regional government has the authority to choose its own regional tax policy. Each regional government behaves non-cooperatively with respect to other regions and maximizes current period regional tax revenue. We abstract entirely from the central government in the second federal arrangement.

The key feature of the second federal arrangement, which is absent in the first, is regional tax competition. An important aspect of the regional government's problem will be to consider an off equilibrium event, namely factory migration to a region with a lower tax rate. Since only regionally symmetric equilibria are considered, we can restrict our attention to the case of a single regional government when all other regional governments choose the same tax policy.

Formally, given, A_{t-1}, X_{t-1}, N_t and the tax policy, $\overline{\tau}_t$, of all other period t regional governments, the regional government chooses τ_t to solve:

$$\max_{\tau_t} \quad \tau_{f,t} \lambda(\tau_t; \overline{\tau}_t) B_f (\Phi(A_{t-1})X_{t-1})^\theta N_{f,t}^{1-\theta} + \tau_{h,t} B_h N_{h,t}^{1-\phi} \tag{4}$$

[9] A subtle but important feature of this specification in this environment is that the period t central government's choice of current period tax rates has no strategic interaction with future central government choices. The current old managers do not consider future central government taxes since they are only concerned with current consumption. Since the young choose their actions first, the central government takes the young's actions as given. This eliminates any influence between central governments of different periods. Consequently, there is no strategic interaction among the big players. It is likely that by considering Markov equilibria in terms of tax policies, the main result of this paper would remain if the sequence of government and young actions were reversed.

[10] Here, let $\Phi(A_0)X_0 \equiv A_1$. Recall A_1 is a parameter that is normalized to one.

s.t.

$$0 \leq \tau_{f,t} \leq 1 \quad \text{and} \quad 0 \leq \tau_{h,t} \leq \overline{\tau} < 1$$

where,

$$\lambda(\tau_t; \overline{\tau}_t) = \begin{cases} 0, if \tau_{f,t} > \overline{\tau}_{f,t} \\ 1, if \tau_{f,t} = \overline{\tau}_{f,t} \\ \tilde{\lambda}, otherwise. \end{cases}$$

In the region considered, the measure of factories $\lambda(\tau_t; \overline{\tau}_t)$, depends on both the region's tax policy τ_t, the tax policy of other regions $\overline{\tau}_t$, and many other variables.[11] If $\tau_{f,t} > \overline{\tau}_{f,t}$, all factories in the region move to other regions to produce at the lower factory tax rate $\overline{\tau}_{f,t}$. If $\tau_{f,t} = \overline{\tau}_{f,t}$, then factories stay and factory output is the same in all regions. If $\tau_{f,t} < \overline{\tau}_{f,t}$, then factories from other regions will move into the region until the maximum regional population density is reached and total factory output in the region will be $\tilde{\lambda} B_f (\Phi(A_{t-1}) X_{t-1})^{\theta} N_{f,t}^{1-\theta}$.[12] The factory migrations can be modeled formally by a lottery that has the property that no other region's variables or the economy-wide variables are affected.[13]

Definition F2. Under the second federal arrangement, a symmetric equilibrium is a sequence of tax policies $\tau = \{\tau^t\}_{t=1}^{\infty}$ and a non-negative sequence $C_1^0, \{A_{t+1}, W_t, X_t, E_t, N_t, C^t\}_{t=1}^{\infty}$ such that the later is a τ-equilibrium and such that for every $t \geq 1$, given A_{t-1}, X_{t-1}, N_t and the tax policy in other regions, $\overline{\tau}_t$, the regional government's choice of τ_t solves (4) and $\tau_t = \overline{\tau}_t$.

4 Main result: equilibrium existence and characterization

Theorem – Main result. *There exists a non-empty, open set of parameters, Ω, such that for any economy $\mu \in \Omega$, both federal arrangements admit the existence of a symmetric equilibrium. In addition, all symmetric equilibria are characterized as follows*:

(F1) *For any symmetric equilibrium under the first federal arrangement for μ, the following three properties hold for all $t \geq 1$*:

[11] Implicitly embedded in this measure is the best response of factory managers given the tax policies, τ_t and $\overline{\tau}_t$. Note that since only symmetric equilibria are considered, all factories will be the same. This feature is exploited in the expression for the regional factory tax revenue. Note that there is one situation for which this formulation is not completely satisfactory. Consider the case where one region undercuts all the other regions' factory tax rate. Implicitly, there is a lottery for the chance to migrate since there is a population limit. Those afforded the opportunity may, when young, invest more in human capital than all other young. Off equilibrium events such as this could be handled by a more general functional form for factory tax revenue, which would depend on the same variables but would not include the expression of the product of a measure and the typical factory output. This type of factory output function could express the region's factory output for all such off equilibrium events. To keep notation to a minimum in this paper, the more general form of the regional factory output, which would be a function of τ^t and many other variables, is not given.

[12] The region's total factory output will be greater than that of other regions and will depend on many variables.

[13] This lottery is described in the Part 4 of the proof of the main result.

F1-1. $\tau_{t+1} = (1, \overline{\tau})$,
F1-2. $X_{s,t} = 0$,
F1-3. $A_{t+1} = 0$ and $Y_t = B_h(1+\varepsilon)^{1-\phi}$,

(F2) *For any symmetric equilibrium under the second federal arrangement for* μ*, the following three properties hold for all* $t \geq 1$:
F2-1. $\tau_{t+1} = (0, \overline{\tau})$,
F2-2. $X_{s,t} > 0$,
F2-3. $\lim_{t\to\infty} A_t = \infty$ and $\lim_{t\to\infty} Y_t = \infty$.

Proof. See Appendix.

The intuition behind this result is simple. The key to growth is the increase of the stock of knowledge, which is determined by individual human capital investment decisions. Consider a young person deciding whether to invest in human capital today. The expected future return on this investment is crucial. Under the first federal arrangement, the young person knows that in the future he will have nowhere to run. Therefore, the central government will expropriate the returns of his investment. Consequently, he does not invest in human capital, the aggregate stock of knowledge remains constant and stagnation results. Under the second federal arrangement, the young person knows that in the future he will be able to move to another region if any regional government raises its tax rate. Moreover, he knows that tax competition for factory tax revenue will then drive the factory tax rates to zero. Consequently, he invests in human capital, the aggregate stock of knowledge increases and growth results.

5 Conclusion

This paper introduces two federal arrangements into a neoclassical growth model with many regions and a mobile factor, human capital, which is the key to growth. Under the first federal arrangement the central government chooses a uniform tax policy for all regions, whereas under the second, each regional government non-cooperatively chooses its own tax policy. For both federal arrangements, equilibrium is shown to exist. Under the first federal arrangement, equilibrium is characterized by high tax rates, no human capital investment and stagnation. Under the second federal arrangement, equilibrium is characterized by low tax rates, positive human capital investment and growth. This result stems from a time consistency problem. The lack of tax competition in the first federal arrangement forces a time consistency problem on the central government so that the state is unable to commit not to expropriate in the future. In contrast, regional tax competition in the second federal arrangement acts as a commitment device and allows the state to commit not to expropriate in the future. In this particular environment, the devolution of tax policy authority from the central government to the regional governments introduces regional tax competition, which removes the time consistency problem, and commits the state not to expropriate in the future. It must be emphasized that this is only a theoretical argument and that its relevance to the process of long run economic growth requires an analysis of evidence.

This paper contributes to an emerging body of literature in growth theory (particularly Acemoglu and Robinson, 2000, 2002) that emphasizes the role of government in long run economic performance. An interesting extension of the growth model with federal arrangements presented here would be to formally introduce government officials into the model by giving them utility functions. Government could be formally modeled as a dynamic coalition including both young and old officials. A dynamic problem for the government would allow the old officials to set tax rates to maximize their utility subject to a participation constraint, where the young officials were offered a consumption plan that gave a lifetime utility level at least that of other career options.

This paper shows that the relationship between the state and long run economic performance can be analyzed by introducing game theoretic elements into a standard neoclassical growth model in a tractable way. This relationship can also be analyzed by considering how different structures of authority within the state affect the interactions between government officials and vested interest groups in the economy. When vested interest groups seek to influence government policy, different structures of authority within the state will give rise to different equilibrium policy choices in response. Exploration of the various ways the structure of authority within the state affects the government's equilibrium economic policy choice and property right enforcement is an important and promising direction of future research in economic development.

Appendix

The proof of the main result is broken into four parts. A subspace of parameters is defined that is non-empty and open with respect to the standard Euclidean subspace topology. Parts 1 and 2 prove existence and characterize symmetric equilibria under the first federal arrangement (F1), and similarly for Parts 3 and 4 under the second federal arrangement (F2).

Define our non-empty, open set of parameters $P \subseteq R_+^{12}$ as

$$\begin{aligned} P \equiv \{(\overline{L}, A_1, M, \theta, B_f, B_h, \phi, \gamma, \overline{\tau}, \varepsilon, \omega, \beta) \in R_+^{12} | \overline{L} \\ = 1, A_1 = 1, M > 4, 1 > \theta > 0, \\ B_f > 0, B_h > 0, 1 > \phi > 0, \gamma > 1, 1 > \overline{\tau} > 0, 1 > \varepsilon, \omega > 0, \beta > 0\}. \end{aligned}$$

Part (1). Existence under F1. This is a constructive existence proof. Fix $\mu \in P$. For all $t \geq 1$ set $A_{t+1} = 0, \tau_t = \overline{\tau}_t = (1, \overline{\tau}), W_{h,t} = (1-\phi)(1-\overline{\tau})B_h(1+\varepsilon)^{-\phi}, W_{f,t} = X_{s,t} = X_{f,t} = E_{f,t} = N_{f,t} = 0, C_{t+1}^t = (1-\overline{\tau})B_h(1+\varepsilon)^{1-\phi} - W_{h,t+1} + \omega, N_{h,t} - \varepsilon = X_{h,t} = 1, C_1^0 = (1-\overline{\tau})B_h(1+\varepsilon)^{1-\phi} - W_{h,1} + \omega, E_{h,t} = \varepsilon$, and $C_t^t = W_{h,t} + \omega$. It is easy to show that this sequence satisfies definition F1. □

Part (2). Characterization under F1. Fix any $\mu \in P$ and consider any symmetric equilibrium under the first federal arrangement, $\tau = \{\tau^t\}_{t=1}^{\infty}$ and $C_1^0, \{A_{t+1}, W_t, X_t, E_t, N_t, C^t\}_{t=1}^{\infty}$.

For all $t \geq 1, \tau_t = (1, \overline{\tau})$: Since labor supply and demand were determined before policy was chosen, household tax rates must be maximal. It follows that if some factory tax rate was not maximal, factory output must be positive. Since all regions have the same factory tax rate, the previous argument also applies to factory tax rates. Thus tax rates must be maximal for all $t \geq 1$.

For all $t \geq 1, X_{s,t} = 0$: Suppose by way of contradiction that for some $t \geq 1, X_{s,t} > 0$. Since factory tax rates are maximal it is easy to show that generation t could have increased its lifetime utility by sending $X_{s,t}$ to work for a strictly positive household wage. Thus $X_{s,t} = 0$ for all $t \geq 1$.

For all $t \geq 1, A_{t+1} = 0$ *and* $Y_t = B_h(1+\varepsilon)^{1-\phi}$: Follows from $X_{s,t} = 0$ for all $t \geq 1$. □

Part (3). Existence under F2. This proof follows standard pure exchange overlapping generations equilibrium existence proofs, such as in Balasko and Shell (1980), with some modifications to accommodate both production and an externality. Fix any $\mu \in P$. The existence proof is presented in five steps. Step 1 introduces a modified economy and its equilibrium concept. Step 2 constructs a set, Δ, that is compact in the product topology and contains all equilibria for the modified economy. Step 3 defines and proves the existence of a T-period truncated, modified economy. In this key step, a modification to the standard existence proof will be introduced in order to accommodate the learning externality. Step 4 exploits the compactness of Δ to find a limit of a sequence of truncated economies and shows that the limit is an equilibrium for the modified economy. Step 5 uses the equilibrium for the modified economy to produce the desired symmetric equilibrium.

Step 1) Introduce a modified economy and its definition of equilibrium.

First, set $\tau_t = \overline{\tau}_t = (0, \overline{\tau})$ for all $t \geq 1$. Before introducing the modified utility functions, it will be helpful to first bound equilibrium wages. Consider only wages where $W_{f,t} = W_{h,t} = W_t > 0$ for all $t \geq 1$. An equilibrium with such wages will be produced. We can now simplify the analysis by abstracting from the splits in labor supply decisions. The young's labor supply will be $(1 - X_{s,t})$ and the old will inelastically supply ε units of labor services. Since wages in the two sectors are equal, any split that gives rise to the total labor supply will be an equilibrium, i.e. there will be a continuum of labor splits for the equilibrium total labor supply. Later, we will produce one split that gives rise to the equilibrium total labor supply. Next we bound equilibrium wages.

Lemma 3.1. *Given* $\mu \in P$, *there exists* $\overline{W} > \underline{W} > 0$, *such that for any feasible stocks of knowledge, if* $W_t = W_{f,t} = W_{h,t} > 0$ *and the period t labor market clears, it must be that*

$$\underline{\mathrm{W}} \leq W_t \quad \text{and} \quad W_t \leq \overline{W}_t \equiv \gamma^t \overline{W} \ .$$

Proof. Using household and factory labor demands, the inelastic labor supply of the old, ε, and the maximum labor supply M, it is straightforward to derive the desired $\overline{W} > \underline{W} > 0$, (which do not depend on β) from labor market clearing conditions. □

We next introduce a modified economy. The utility functions and the consumption sets will be modified and then modified utility maximization problems will be presented.

It shall be helpful to load production into the utility function so that a linear Arrow-Debreu budget constraint can be used. Define,

$$\tilde{U}_t(c_t, k_t, x_t, n_{f,t+1}, n_{h,t+1})$$
$$\equiv \log c_t + \beta \log[(1-\overline{\tau})B_h n_{h,t+1}^{1-\phi} + B_f(\Phi(A_t)x_t)^\theta n_{f,t+1}^{1-\theta} + k_t],$$

where c_t is consumption when young, x_t is the measure of young sent to learn, and where $n_{f,t+1}$ and $n_{h,t+1}$ are the labor demands for the factory and household firms respectively. Lastly, the sum of after tax production and k_t is consumption when old. Restrictions will be imposed on consumption sets to guarantee that only strictly positive arguments enter the log functions.

Next, we produce a modified utility function and consumption set for each generation. Consider the standard utility function $\log x + \beta \log y$ defined on and the closed, convex upper contour set $U_\omega = \{(x,y) \in R^2_{++} | \log x + \beta \log y \geq \log \omega + \beta \log \omega\}$. In any equilibrium the consumption plan of any generation will be an element of U_ω, since everyone has an endowment (ω, ω) of the consumption good. In addition we can bound period t consumption by feasible aggregate production. For all $t \geq 1$, let $\overline{Y}_t = B_h M^{1-\phi} + B_f(\gamma^{t+1})^\theta M^{1-\theta} + 2\omega$ and $U^t_\omega = U_\omega \cap \{(x,y) \in R^2_{++} | x, y \leq \overline{Y}_t\}$. Any equilibrium consumption plan clearly must satisfy, $(C^t_t, C^t_{t+1}) \in U^t_\omega$. Therefore there exists a strictly positive vector, $(\underline{c}^t_t, \underline{c}^t_{t+1}) \in R^2_{++}$, such that the set $\{(x,y) \in R^2_{++} | x \geq \underline{c}^t_t, y \geq \underline{c}^t_{t+1}\}$ contains the set U^t_ω in its interior. Now let $\underline{u}_t = \log \underline{c}^t_t + \beta \log \underline{c}^t_{t+1}$ and define our modified utility function for generation t as:

$$U_t(c_t, k_t, x_t, n_{f,t+1}, n_{h,t+1}) \equiv \tilde{U}_t(c_t, k_t, x_t, n_{f,t+1}, n_{h,t+1}) - \underline{u}_t.$$

The generation t consumption set is defined as:

$$Z^t \equiv \{(c_t, k_t, x_t, n_{f,t+1}, n_{h,t+1}) \in R^5 | c_t \geq \underline{c}^t_t, 1 \geq x_t \geq 0, n_{f,t+1}, n_{h,t+1} \geq 0$$

and

$$k_t \geq \underline{k}_t(c_t, x_t, n_{f,t+1}, n_{h,t+1})\},$$

where,

$$\underline{k}_t(c_t, x_t, n_{f,t+1}, n_{h,t+1})$$
$$\equiv \underline{c}^t_{t+1} - \min\{\overline{Y}_{t+1}, (1-\overline{\tau})B_h n_{h,t+1}^{1-\phi} + B_f(\Phi(A_t)x_t)^\theta n_{f,t+1}^{1-\theta}\}.$$

This formulation of the consumption set is standard with the exception of the lower bound on k_t. The lower bound guarantees that the sum of after tax production and k_t is at least $\underline{c}^t_{t+1} > 0$. The consumption set also guarantees a minimum level for young consumption of $\underline{c}^t_t > 0$, so that the utility function is well defined. Thus by construction, the consumption set is bounded below by $(\underline{c}^t_t, \underline{c}^t_{t+1})$, i.e. $(\underline{c}^t_t, \underline{c}^t_{t+1}) \leq (c^t_t, c^t_{t+1})$ for all $(c^t_t, c^t_{t+1}) \in Z^t$, where $(\underline{c}^t_t, \underline{c}^t_{t+1}) \in R^2_{++}, c^t_t \equiv c_t$

and $c^t_{t+1} \equiv (1-\overline{\tau})B_h n^{1-\phi}_{h,t+1} + B_f(\Phi(A_t)x_t)^\theta n^{1-\theta}_{f,t+1} + k_t$. This fact will be used later to bound consumption good prices. Define the initial old's utility function and consumption set as follows,

$$U_0(k_0, n_{f,1}, n_{h,1}) \equiv \log[(1-\overline{\tau})B_h n^{1-\phi}_{h,1} + B_f A^\theta_1 n^{1-\theta}_{f,1} + k_0] - \log\frac{\omega}{2}$$

and

$$Z^0 \equiv \{(k_0, n_{f,1}, n_{h,1}) \in R^3 | n_{f,1}, n_{h,1} \geq 0$$

and

$$k_0 \geq {}^{\omega}\!/_2 - \min[\overline{Y}_1, (1-\overline{\tau})B_h n^{1-\phi}_{h,1} + B_f A^\theta_1 n^{1-\theta}_{f,1}]\}.$$

Finally note that the restrictions imposed on all consumption sets exclude no equilibrium household allocations, since only equilibrium conditions were used. We next introduce the modified utility maximization problems.

Initial old group's utility maximization problem: Given A_1, p_1 and w_1 the initial old group's problem is given by:

$$\begin{array}{l}\max U_0(k_0, n_{f,1}, n_{h,1}) \\ (k_0, n_{f,1}, n_{h,1}) \in Z^0\end{array} \tag{5}$$

s.t.

$$p_1 k_0 \leq w_1(\varepsilon - n_{f,1} - n_{h,1}) + p_1\omega.$$

Generation t young group's utility maximization problem: Given A_t, p_t, p_{t+1}, w_t and w_{t+1} the generation t young group's problem is given by:

$$\begin{array}{l}\max U_t(c_t, k_t, x_t, n_{f,t+1}, n_{h,t+1}) \\ (c_t, k_t, x_t, n_{f,t+1}, n_{h,t+1}) \in Z^t\end{array} \tag{6}$$

s.t.

$$p_t c_t + p_{t+1}k_t \leq w_t(1-x_t) + w_{t+1}(\varepsilon - n_{f,t+1} - n_{h,t+1}) + (p_t + p_{t+1})\omega\ .$$

Note that the profit maximization problems are embedded in the modified utility maximization problems (5) and (6). In their essentials, these modified utility maximization problems resemble standard utility maximization problems for a pure exchange overlapping generations economy with two goods in each period. The consumption sets, Z^t, Z^0 are all closed, convex and bounded below. In addition the utility functions U_t, U_0 are well defined on Z^t, Z^0 respectively, non-negative, C^2, strictly increasing in all their arguments, and strictly concave. Furthermore, each generation has a linear Arrow-Debreu budget constraint. Note that each generation is endowed with a strictly positive amount of each good, so there is an element in the interior of the consumption set that is affordable at any non-negative set of prices. Next, equilibrium good and labor market clearing conditions are presented.

Period t consumption good and labor market clearing:

$$c_t + k_{t-1} = 2\omega \quad \text{and} \quad n_{f,t} + n_{h,t} = (1 - x_t) + \varepsilon \ .$$

Recall k_{t-1} is generation $t-1$ consumption when old less period t after tax production.

Definition ME. For the modified economy associated with μ, an equilibrium is a non-negative sequence $\{A_{t+1}, p_t, w_t, c_t, k_{t-1}, x_t, n_{f,t}, n_{h,t}\}_{t=1}^{\infty}$ such that the following four conditions hold:

1. Given A_1, p_1 and w_1, (5) is solved by $k_0, n_{f,1}, n_{h,1}$,
2. For $t \geq 1$, given A_t, p_t, p_{t+1}, w_t and w_{t+1}, (6) is solved by $c_t, k_t, x_t, n_{f,t+1}, n_{h,t+1}$,
3. The consumption good and labor markets clear in all periods,
4. Consistency of the law of motion, i.e. for all $t \geq 1$, $A_{t+1} = \Phi(A_t)x_t$.

Step 2) Construct a compact set Δ that contains all equilibria of the modified economy.

The desired set, Δ, will be a product of sets that contain elements of the form $\{A_{t+1}, p_t, w_t, c_t, k_{t-1}, x_t, n_{f,t}, n_{h,t}\}_{t=1}^{\infty}$. The key to guaranteeing compactness in the product topology will be to bound each element of the sequence in a compact interval.

We first consider consumption good prices, $\{p_t\}_{t=1}^{\infty}$. The construction of a closed and bounded interval that contains each equilibrium p_t will be outlined here. Consider the utility maximization problem of any young generation t where $p_t = 1$. It can be shown from the necessary first order condition involving t and $t+1$ consumption and the fact that any equilibrium consumption plan must lie in the previously defined set U_ω^t that any equilibrium p_{t+1} admits strictly positive lower and upper bounds. Normalize prices so that $p_1 = 1$. It is straightforward to use a product of t of these upper and lower bounds to construct an interval, $[\underline{p}_t, \overline{p}_t] \subseteq R_{++}$, which contains all equilibrium p_t. This is a standard technique used to bound equilibrium prices in overlapping generation models (i.e. Balasko and Shell, 1980, pp. 287–288). The infinite Cartesian product of these intervals contains all equilibrium prices, $\{p_t\}_{t=1}^{\infty}$. Similarly, using these bounds for consumption good prices and the real wage bounds from Lemma 3.1, we can produce similar wage intervals, the product of which contains all equilibrium wages, $\{w_t\}_{t=1}^{\infty}$. Using the definition of the law of motion for knowledge, and minimum and maximum values for the supply of consumption and labor goods, we can produce similar products for the other seven components of an equilibrium. The product of these eight infinite products will be defined as the desired set, Δ. By construction any equilibrium for the modified economy will satisfy, $\{A_{t+1}, p_t, w_t, c_t, k_{t-1}, x_t, n_{f,t}, n_{h,t}\}_{t=1}^{\infty} \in \Delta$. Since Δ is the Cartesian product of a collection of compact topological spaces, by Tychonoff's theorem, Δ is compact in the product topology (see Kelly, 1991, p. 143).

Step 3) Define and prove the existence of an equilibrium for a truncated, modified economy.

Fix $T \geq 2$ and we will construct a T-period truncated, modified economy. There are two differences between the T-period economy and the modified economy. First the terminal generation T young group's utility maximization problem is modified as follows. Since effectively there is no period $T+1$ in this truncated economy, we set $x_T = 1/2$.

Given p_T and w_T the terminal young group's problem is given by:

$$\max \log c_T - \log \frac{\omega}{2} \tag{7}$$

$$(c_T, x_T) \in Z^{YT} = \{(x, y) \in R^2_{++} | x \geq \omega/2, 1 \geq y \geq 1/2\}$$

s.t.

$$p_T c_T \leq w_T(1 - x_T) + p_T \omega.$$

The second difference is that the T-period equilibrium is truncated. That is $\{A_{t+1}, p_t, w_t, c_t, k_{t-1}, x_t, n_{f,t}, n_{h,t}\}_{t=1}^T$ is a truncated sequence such that condition 2 in definition ME holds for all $1 \leq t \leq T-1$. The initial old and terminal young both solve their problems, i.e. problems (5) and (7) are solved. Also for all $1 \leq t \leq T$, the consumption good and labor markets clear. Lastly, $A_{t+1} = \Phi(A_t)x_t$ for $1 \leq t \leq T$. Any sequence, $\{A_{t+1}, p_t, w_t, c_t, k_{t-1}, x_t, n_{f,t}, n_{h,t}\}_{t=1}^T$ that satisfies the above conditions is called an equilibrium for the T-period truncated, modified economy.

Lemma 3.2. *For all $T \geq 2$, there exists an equilibrium for the T-period truncated, modified economy.*

Proof. This is the key step in the proof of existence under F2. The proof closely follows the standard techniques used to prove existence for a truncated T-period pure exchange overlapping generations economy, since the modified economy meets almost all the technical conditions of a standard pure exchange economy. The utility functions are non-negative, C^2, strictly increasing in all their arguments, and strictly concave. The consumption sets are closed, convex, and bounded below and all utility maximization problems have a standard linear Arrow-Debreu budget constraint. Since each generation is endowed with a strictly positive amount of all consumption and labor goods throughout their lives, the interior of their consumption set is non-empty and their endowments are affordable under all non-negative prices. Given a fixed $\{A_{t+1}\}_{t=1}^T$, it is straightforward to show that a compensated equilibrium for this economy exists. It also is easy to establish that every household is indirectly resource related to every other household, as in Arrow and Hahn (1971), Ch. 5. Therefore the compensated equilibrium gives rise to the existence of a competitive equilibrium for this economy. The one condition not satisfied by the standard proof is the consistency of the law of motion for knowledge stocks. This is an externality that the standard proof does not handle. We can slightly modify the proof of existence in the key step in order to guarantee that this condition is satisfied.

In what follows, an outline of a modification of the correspondence used in Arrow and Hahn's proof of existence of a compensated equilibrium is presented. The modification will imply that a fixed point resulting from an application of Kakutani's Fixed Point Theorem will produce a desired equilibrium. The correspondence will be briefly introduced and the modification will be outlined.

Arrow and Hahn define a correspondence, $\Psi : P \times U \times X \to P \times U \times X$, as:

$$\Psi(p,u,x) = \Psi^P(p,u,x) \times \Psi^U(p,u,x) \times \Psi^X(p,u,x),$$

where generally speaking the sets P, U and X correspond to sets of prices, utility levels and allocations. It is shown that $P \times U \times X$ is a compact, convex subset of a finite dimensional Euclidean space, that $\Psi : P \times U \times X \to P \times U \times X$ is upper hemi-continuous, and that $\Psi(p,u,x)$ is non-empty and convex valued for all $(p,u,x) \in P \times U \times X$. Kakutani's Fixed Point Theorem is applied and a fixed point gives rise to a compensated equilibrium (Arrow and Hahn, 1971, pp. 114–116).

The modification needed adds a fourth set to the domain and a fourth component to the correspondence. The fourth set, S^T, contains all feasible sequences of truncated stocks of knowledge, $A^T \equiv \{A_{t+1}\}_{t=1}^{T} \in S^T$, where S^T is the Cartesian product of the first T bounding intervals for the stocks of knowledge in the construction of Δ, which was outlined in Step 2. Define the new domain by $P \times U \times X \times S^T$. We can naturally extend the original three components of the correspondence Ψ to correspondences that formally depend on A^T, $\tilde{\Psi}^P$, $\tilde{\Psi}^U$ and $\tilde{\Psi}^X$. Note that since the stocks of knowledge enter utility functions and consumption sets in a continuous fashion, $\tilde{\Psi}^P$, $\tilde{\Psi}^U$ and $\tilde{\Psi}^X$ all retain their desired properties [a similar extension is outlined by Arrow and Hahn (1971), pp. 132–136]. The fourth component of the new correspondence will have the property that a fixed point implies the consistency of the law of motion. Consider the natural cost minimization problem associated with the utility maximization in (6). Let $X_t(p,u,x,A^T)$ denote the compensated demand correspondence for the variable x_t. It is straightforward to show that this correspondence is non-empty and convex valued. Since the utility function is continuous and strictly concave, the standard proof by contradiction shows that it is also upper hemi-continuous. Now define:

$$\tilde{\Psi}^S(p,u,x,A^T) \equiv \{\tilde{A}^T \in S^T \,|\, \tilde{A}_{t+1} = \Phi(A_t)x_t \quad \text{for some } x_t \in X_t(p,u,x,A^T)\} ,$$

for all $(p,u,x,A^T) \in P \times U \times X \times S^T$. Since all the compensated demand functions are non-empty, convex valued and upper hemi-continuous, $\tilde{\Psi}^S$ inherits all these properties.

Finally, define $\tilde{\Psi} : P \times U \times X \times S^T \to P \times U \times X \times S^T$ by:

$$\begin{aligned}\tilde{\Psi}(p,u,x,A^T) = {} & \tilde{\Psi}^P(p,u,x,A^T) \times \tilde{\Psi}^U(p,u,x,A^T) \\ & \times \tilde{\Psi}^X(p,u,x,A^T) \times \tilde{\Psi}^S(p,u,x,A^T) .\end{aligned}$$

Therefore the set $P \times U \times X \times S^T$ is a compact subset of a finite dimensional Euclidean space and the correspondence $\tilde{\Psi} : P \times U \times X \times S^T \to P \times U \times X \times S^T$ is non-empty, convex valued and upper hemi-continuous. By Kakutani's Fixed Point Theorem there exists a fixed point, i.e. $(\hat{p},\hat{u},\hat{x},\hat{A}^T) \in \tilde{\Psi}(\hat{p},\hat{u},\hat{x},\hat{A}^T)$. Note $\hat{A}^T \in \tilde{\Psi}^S(\hat{p},\hat{u},\hat{x},\hat{A}^T)$ implies the consistency condition for the law of motion.

Thus a compensated demand exists, where the law of motion for knowledge stocks is consistent. Since every household is indirectly resource related to every other household, this compensated equilibrium gives rise to a competitive equilibrium for the T-period truncated, modified economy. □

Step 4) Existence of an equilibrium for the modified economy.

By Lemma 3.2 for all $T \geq 2$, there exists an equilibrium for the T-period truncated, modified economy, $\{A_{t+1}, p_t, w_t, c_t, k_{t-1}, x_t, n_{f,t}, n_{h,t}\}_{t=1}^{T}$. It is easy to affix a tail onto this sequence so that the resulting infinite sequence satisfies $\delta_T \in \varDelta$ and the first $T-1$ components satisfy the definition of equilibrium for the modified economy presented in Step 1. Since $\varDelta$ is compact in the product topology, the sequence $\{\delta_T\}_{T=2}^{\infty}$ admits a convergent subsequence with limit in $\varDelta$, i.e. there is some $\{\hat{\delta}_n\}_{n=2}^{\infty} \subseteq \{\delta_T\}_{T=2}^{\infty}$ and $\hat{\delta} \in \varDelta$, with $\lim_{n\to\infty} \hat{\delta}_n = \hat{\delta}$. Since each single equilibrium condition in definition ME only involves a finite number of variables, each of the equilibrium conditions is continuous in the product topology. Since $\lim_{n\to\infty} \hat{\delta}_n = \hat{\delta}$ with respect to the product topology, the continuity of the equilibrium conditions in the product topology imply the equilibrium conditions also hold for the limit, $\hat{\delta}$. Thus $\hat{\delta}$ is an equilibrium for the modified economy.

Step 5) Existence of a symmetric equilibrium.

It is straightforward to use an equilibrium from the modified economy to naturally define a candidate-symmetric equilibrium under the second federal arrangement. Definition ME conditions 1, 3 and 4 imply conditions 1, 3 and 4 in definition T (τ-equilibrium). We need to establish condition 2 in definition T. One can manipulate the first order Kuhn-Tucker conditions and Lagrange multipliers to show that necessary conditions from utility maximization in (6) imply sufficient conditions for maximization of (2). Thus condition 2 is satisfied, so that we have a τ-equilibrium. Lastly, it is easy to show that $\tau_t = (0, \overline{\tau})$ solves the regional government's problem, (4), for all $t \geq 1$, so that definition F2 is satisfied. Thus a symmetric equilibrium exists under the second federal arrangement. □

Part (4) Characterization under F2. Clearly, household tax rates are maximal. The main difficulty lies in proving factory tax rates are zero. Since an implication of zero factory tax rates is $X_{s,t} \geq \underline{X} > 0$ for all $t \geq 1$, the stocks of knowledge and aggregate output grow without bound.

We shall prove $\overline{\tau}_{f,t+1} = 0$ for all $t \geq 1$ by the method of proof by contradiction (note $\tau_{f,t+1} = \overline{\tau}_{f,t+1}$ by definition F2). Assume for some $t \geq 1$ we have $\overline{\tau}_{f,t+1} > 0$ for some symmetric equilibrium under F2. This implies that tax revenue could not have been maximized for such a positive factory tax rate in a symmetric equilibrium under F2. In particular we will show that one region can cut the tax rate in half, induce factories to move into the region and increase its tax revenue. This will involve the use of a lottery. Lotteries in this model environment select households from other regions in the previous period and offer the option of moving to the region and paying taxes at half of the rate in other regions. If the symmetric equilibrium is such that there are factories in the other regions, all such households will want to move. In this case, the migration of factories until the population limit is reached will more than double factory output, creating a net increase in factory tax revenue. This is a contradiction. If there are no factories in the current period and if preferences are sufficiently patient, the lottery-winning households will accept the offer, send some young members to school and move when old. In this case, the factory tax revenue will clearly increase. Thus we have a contradiction. Note that lotteries can be administered so that no variable in any region is affected: we can pick one household from each region and the result follows from properties of real numbers. It remains to be shown that for the symmetric equilibrium where there are no factories in the current period, households of the previous period are sufficiently patient.

When factory output is zero in period $t+1$, a lottery will be held in period t and the young household winners will be announced before their decisions are made. In this case they have the opportunity to send some young to learn and to have their factories move to the region when old to produce at the factory tax rate $\overline{\tau}_{f,t+1}/2$. To ensure that the young household will accept such an offer they must be sufficiently patient. Note that in any symmetric equilibrium labor market clearing implies $W_{h,t} \geq W_{f,t}$, so that workers will earn $W_{h,t}$. Consider the first order necessary condition in the young's utility maximization problem (2), where we have substituted consumption when young and old with the values in terms of income:

$$\frac{W_{h,t}}{W_{h,t}(1-X_{s,t})+\omega} = \beta\frac{\theta(1-\theta)^{(1-\theta)/\theta}(1/2)B_f^{1/\theta}(W_{f,t+1})^{(\theta-1)/\theta}\Phi(A_t)}{\pi_h(W_{h,t+1})+\pi_f(X_{s,t},W_{f,t+1})+\varepsilon W_{h,t+1}+\omega}. \quad (8)$$

We wish to impose a condition on β that implies LHS (8) < RHS (8) for all $t \geq 1$ and for all $X_{s,t} < \underline{X}$, for some $1 \geq \underline{X} > 1/\gamma$. This will imply any reduced factory tax rate (note $\overline{\tau}_{f,t+1}/2 \leq 1/2$) will be accepted by the young since (8) can hold only if $X_{s,t} \geq \underline{X}$. It is straightforward to bound the LHS (8) above by a term, K^{LHS}, which depends on parameters other than β and t. Similarly, using wage bounds from Lemma 3.1 (in the proof of Part 3) we can bound the RHS (8) from below by an expression of the form $\beta \cdot K^{RHS}$, where the later term depends on terms other than β and t. Thus when we restrict our attention to $\beta \geq K^{LHS}/K^{RHS}$, for any $t \geq 1$, (8) implies $X_{s,t} \geq \underline{X} > 1/\gamma$. Any such generation would accept such an offer to move into an undercutting tax region. Since factory output is zero in the case under consideration here, factory tax revenue would increase for the undercutting region in period $t+1$.

Consider any $\mu \in \Omega \equiv \{\eta \in P | \beta \geq K^{LHS}/K^{RHS}\}$. For any symmetric equilibrium, the above arguments imply $\tau_t = \overline{\tau}_t = (0, \overline{\tau})$ and that $X_{s,t} \geq \underline{X} > 1/\gamma$ for all $t \geq 1$. It is easy to show the later condition implies both A_t and aggregate output grow without bound. □

Proof of Theorem. Clearly, $\Omega \subseteq P$ is non-empty and open with respect to the subspace topology of R^{12}. The proofs of Parts 1 through 4 are valid for any $\mu \in \Omega$ and imply the main result. □

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